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THE EFFECT OF CONJUGATED AND RADIANT HEAT EXCHANGE ON THE PROCESS OF NON-STATIONARY COMBUSTION OF THE PRODUCTS OF INTENSE GASIFICATION OF A SOLID IN A STREAM OF GAS*

V.M. AGRANAT AND D.A. GUBIN

This paper develops further the results obtained in /1-4/ and uses the approximate mathematical model /2/ of the combustion of the products of intense gasification of the neighbourhood of the leading stagnation point of the body to analyse the effect of the conjugation parameters on the heat exchange, radiation and other factors on the conditions of uniqueness and stability of the stationary combustion modes. When the gasification is carried out at a constant mass flow rate, an analogy is established, depending on the relations between the parameters of the problem, between the model in question and the models of a homogeneous chemical continuous action reactor with a fluidized catalyst layer, and a reactor with a temperature regulator /5/. Simple necessary conditions for the instability of the stationary modes and the appearance of self-excited oscillations are obtained. A strong stabilizing influence of the conjugated heat exchange and intense injection on the combustion

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process is established, and a destabilizing influence of radiant heat exchange is found.

1. Formulation of the problem. Let us consider the flow of high enthalpy gas past the leading stagnation point of a spherically blunted body undergoing gasification. We assume that a strong injection of gasification products occurs, and that we can neglect near the wall the molecular processes of the transfer of momentum, mass and energy, as compared with convective transfer. We assume that the gas phase reaction rate depends only on the temperature and concentration of a single limiting component of the gaseous mixture formed in the course of gasification. Then, according to /1, 2/ the problem of determining the conditions of existence, uniqueness and stability of the stationary modes of heat and mass exchange between the neighbourhood of the stagnation point of the streamlined body and the gas flow, reduces mathematically to determining the conditions for the existence, uniqueness and stability of the stationary solutions of the following boundary value problem written in dimensionless form:

$$ff'' = \frac{1}{2}\left[(f')^2 - \frac{\rho_e}{\rho}\right], \quad \frac{\rho_e}{\rho} = \frac{1+\beta\Theta}{1+\beta\Theta_e}\frac{M_e}{M} \tag{1.1}$$

$$f\Theta' = \frac{1}{\pi_t}\left[\frac{\partial\Theta}{\partial\tau} - \frac{1}{\gamma}\pi_q\pi_\delta R_2(C,\Theta)\right]$$

$$fC' = \frac{1}{\pi_t}\left[\frac{\partial C}{\partial\tau} + \pi_\delta R_2(C,\Theta)\right] \tag{1.2}$$

$$\frac{\partial^2\Theta_s}{\partial y_s^2} + \gamma\sqrt{\pi_\varkappa}\frac{\rho_w}{\rho_s}R_1\frac{\partial\Theta_s}{\partial y_s} = \frac{\partial\Theta_s}{\partial\tau}$$

$$\frac{\lambda_w\rho_w}{\lambda_e\rho_e}\sqrt{\frac{\pi_t}{\mathrm{Pr}}}\left(\frac{\partial\Theta}{\partial\eta}\right)_w + \alpha_1\frac{\rho_w}{\rho_e}R_1 -$$

$$-\pi_\sigma[(1+\beta\Theta_w)^4 - (1+\beta\Theta_e)^4] = -K_\varepsilon\left(\frac{\partial\Theta_s}{\partial y_s}\right)_w \tag{1.3}$$

$$-\frac{\lambda_w}{\lambda_e}L\sqrt{\frac{\pi_t}{\mathrm{Pr}}}\left(\frac{\partial C}{\partial\eta}\right)_w = \gamma R_1(1-C_w)$$

$$\Theta|_{\eta=0} = \Theta_s|_{y_s=0}, \quad \Theta_s|_{y_s\to\infty} = -\Theta_{sH}$$

$$f|_{\eta=0} = f_w = -\frac{\gamma R_1}{\sqrt{\pi_t \mathrm{Pr}}}\frac{\rho_w}{\rho_e}, \quad \left(\frac{\partial f}{\partial\eta}\right)_{\eta=0} = 0$$

$$\tau = \frac{t}{t_*}, \quad t_* = \frac{2y_*^2\rho_e c_{pe}}{\lambda_e}, \quad y_* = \frac{\lambda_e R T_*^2}{\rho_e E_1 |q_1| k_1}\exp\frac{E_1}{RT_*}$$

$$y_s = -\frac{y\sqrt{\pi_\varkappa}}{y_*}, \quad \pi_\varkappa = \frac{\lambda_e\rho_s c_{ps}}{\lambda_s\rho_e c_{pe}}, \quad \Theta = \frac{E_1}{RT_*^2}(T - T_*),$$

$$R_1 = \frac{(\rho v)_w}{\rho_w k_1}\exp\frac{E_1}{RT_*}, \quad \gamma = \frac{c_{pe}RT_*^2}{|q_1|E_1}, \quad \pi_t = t_*\beta_x,$$

$$\beta_x = \left(\frac{du_e}{dx}\right)_{x=0}, \quad \pi_\sigma = \frac{\varepsilon_R\sigma T_*^3 y_* E_1}{R\lambda_e}, \quad \varepsilon_R = \left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)^{-1},$$

$$K_\varepsilon = \sqrt{\pi_c\pi_\rho}, \quad \pi_c = \frac{c_{ps}}{c_{pe}}, \quad \pi_\rho = \frac{\rho_s\lambda_s}{\rho_e\lambda_e}, \quad \beta = \frac{RT_*}{E_1},$$

$$\Theta_{sH} = \frac{E_1}{RT_*^2}(T_* - T_{sH}), \quad L = \frac{\mathrm{Pr}}{\mathrm{Sc}}, \quad \mathrm{Pr} = \frac{\mu c_p}{\lambda}, \quad \mathrm{Sc} = \frac{\mu}{\rho D},$$

$$\alpha_1 = \frac{q_1}{|q_1|}, \quad \pi_q = \frac{q_2}{|q_1|}, \quad \pi_\delta = \frac{1}{2}\pi_t\,\mathrm{Dam}, \quad \mathrm{Dam} = \frac{k_2}{\beta_x}\exp\left(-\frac{E_2}{RT_*}\right),$$

$$\eta = \frac{ru_e}{\sqrt{2\zeta}}\int_0^y \rho\,dy, \quad \zeta = \int_0^x \mu_e\rho_e u_e r^2\,dx$$

Here τ, y_s is the dimensionless time and coordinate, ζ, η are the Dorodnitsyn variables in the Liz form, f, θ are the dimensionless stream function and temperature, C is the mass concentration of the rate controlling component, R_1, R_2 are the dimensionless mass rate of gasification and gas phase reaction, respectively, α_1, α_2, β, γ, Dam, π_t, π_c, π_ρ, π_δ, $\pi_\varkappa$, π_σ, π_q, K_ε, L, Pr, Sc, β_1 are dimensionless parameters, x, y are the coordinates of the orthogonal coordinate system attached to the boundary separating the media, r is the radius of transverse curvature of the body, u, v are the gas velocity components, ρ is the density, c_p is the heat capacity at constant pressure, μ is the dynamic viscosity, M is the molecular weight of the gaseous mixture, T is the temperature, D is the effective dissusion coefficient, λ is the thermal conductivity, E_1, q_1, k_1, E_2, q_2, k_2 are the activation energy, thermal effect and the pre-exponent of the gasification reaction and gas phase reaction respectively, ε_R is the reduced degree of blackness of the system /6/, ε_1, ε_2 are the degree of blackness of the gas and the wall respectively, σ is the Stefan-Boltzmann constant, R is the universal gas constant, a prime denotes differentiation with respect to η, and the indices s, H, e, w, $*$ refer, respectively, to the parameters of the condensed phase, the parameters of condensed phase as $y_s \to \infty$, the parameters of gas phase on the external boundary of the boundary layer, the parameters on the boundary separating the media, and to characteristic quantities.

In deriving Eqs.(1.1), describing the heat and mass transfer in the gaseous phase near the leading stagnation point of the solid, we assumed that the gas is optically transparent, the Prandtl and Schmidt numbers and the product of density and viscosity are all constant, the gas mixture behaves as a binary mixture /7/ and the specific heat capacities of the various components are constant and equal to each other. The equations of continuity and motion were regarded as quasistationary.

We find that in the presence of the gas-phase reaction ($R_2 \neq 0$) and injection ($f_w \neq 0$), the temperature and concentration gradients at the boundary separating the media are, in accordance with (1.1) and /8/, non-zero. Thereforee, when writing down the laws of conservation of mass and energy at the boundary separating the media, we have retained in (1.3) the terms characterizing the energy and mass transfer caused by heat conduction and diffusion processes.

Since the aim of this paper is to make a qualitative study of the modes of heat and mass transfer, there is no need to use the actual initial conditions.

To fix our ideas, we shall make additional assumptions about the kinetics of gasification and the gas phase reaction. Let the gas phase reaction be of first order in the limiting component, and obey Arrhenius's law. We assume that the gasification is exothermic ($\alpha_1 = +1$) and is carried out at a constant mass rate ($(\rho v)_w = \mathrm{const}$), λ is a linear function of temperature, and the molecular weight of the mixture is constant. We choose as the characteristic temperature T_* the temperature of the unperturbed gas flow T_e, and this yields $\Theta_e = 0$.

Under the above assumptions we can use the method in /2/ based on repeated integration of Eq.(1.2) in y_s from 0 to ∞, and evaluation of the resulting integral /9/, to reduce the boundary value problem (1.1)-(1.3) to the second-order dynamic system

$$\frac{dC_w}{d\tau_1} = a(1-C_w) - \bar{C}_1 C_w \exp\left(-\frac{E}{\Theta_{1w}}\right) \equiv P_1(C_w, \Theta_w) \tag{1.4}$$

$$\frac{d\Theta_{1w}}{d\tau_1} = \varepsilon\left\{L_1 a(\Theta_0 - \Theta_{1w}) + \bar{C}_1\bar{C}_2 C_w \exp\left(-\frac{E}{\Theta_{1w}}\right) + Q(1-\Theta_{1w}^4)\right\} \equiv Q_1(C_w, \Theta_w)$$

$$\tau_1 = \beta_x t,\quad \Theta_1 = \frac{T_w}{T_e},\quad E = \frac{E_2}{RT_e},\quad \bar{C}_1 = \frac{k_2}{2\beta_x},\quad \bar{C}_2 = \frac{q_2 C_0}{c_p T_e},$$

$$a = \mathrm{Sc}\, f_w^2,\quad L_1 = L\pi_c,\quad \varepsilon = (1+\pi_\rho)^{-1},\quad \Theta_0 = \frac{T_{sH}}{T_e} + \frac{C_3}{\pi_c},\quad C_3 = \frac{q_1}{c_p T_e},$$

$$Q = \frac{q_R}{q_k},\quad q_R = \varepsilon_R \sigma T_e^4,\quad q_k = \frac{T_e\lambda_e}{f_w}\sqrt{\frac{2\beta_x}{\nu_e}}$$

Here a is the effective dimensionless mass exchange coefficient, $\bar{C}_1$ and $\bar{C}_2$ are the first and second Damköhler gasification number for a gas-phase reaction, C_3 is the second Damköhler gasification number, Q is the relative radiant flux, and ν_e is the kinematic viscosity on the external boundary of the boundary layer.

System (1.4) describes the changes, with time, of the relative mass concentration C_w of the intermediate limiting component and of the dimensionless temperature Θ_{1w} at the boundary separating the media. In this manner we reduce the analysis of the possible modes of heat and mass transfer in the problem in question to a qualitative investigation of a dynamic, multiparameter system (1.4).

2. *Analysis of the mathematical model and the physical interpretation of the results.* The dynamic system (1.4) is a generalization of the systems discussed in /3, 10, 11/, and a special case of the system described in /2/. The latter circumstance enables us to use the results of /2/ to study (1.4), such as the principle of non-parity of the equilibrium states, the presence of a cycle without contact and the conditions of mild excitation of selfexcited oscillations, established in /2/ and /5, 12, 13/.

When there is no radiant heat exchange ($Q = 0$), system (1.4) will differ from the model of a non-isothermal homogeneous chemical continuous-action reactor studied in detail in /5/, the gas-phase diffusion model /3/, and the model of heterogeneous combustion in the boundary layer /10/, when the factor ε, is present in the expression for Q_1, which plays the part of the conjugation parameter of the problem. Generally speaking, $0 \leqslant \varepsilon = (1 + (\lambda_s\rho_s)/(\lambda_e\rho_e))^{-1} \leqslant 1$, where the equality signs correspond to the limiting cases of infinite thermal conductivity of the body ($\lambda_s = \infty$) and thermally insulated wall ($\lambda_s = 0$), which do not occur in practice.

In the majority of heat transfer problems of practical importance, ε is a small parameter. Indeed, when the thermal conductivity λ_s is finite we find, as a rule, that $\lambda_s \gtrsim \lambda_e$ and the density ρ_s of the condensed phase at moderate gas pressures will appreciably exceed the density of the gas ρ_e ($\rho_s \gg \rho_e$), and consequently $\varepsilon \ll 1$. If $\varepsilon \ll 1$, then the rates of change of Θ_{1w} and $\bar{C}_w$ will differ significantly from each other since $Q_1/P_1 = O(\varepsilon)$ $(L_1 \sim 1, \bar{C}_2 \sim 1)$. Moreover, $\partial P_1/\partial C_w < 0$. Therefore, when $\varepsilon \ll 1$, we can apply to system (1.4) the method of quasistationary concentrations /5/, according to which the analysis of system (1.1) can be reduced to the analysis of a single differential equation and a single finite relation:

$$\frac{d\Theta_{1w}}{d\tau_1} = Q_1(C_w, \Theta_{1w}), \quad P_1(C_w, \Theta_{1w}) = 0 \tag{2.1}$$

The equation in (2.1) is analogous to the equation of a thermal explosion /12/. It has a set of stationary solutions, although the selfexcited oscillations and oscillatory instability are inadmissible in the dynamic first-order system (2.1) for any single-valued function Q_1 /13/.

The condition $\rho_s \gg \rho_e$ is violated when the gas pressures are of the order of 10^3 atmospheres, in which case $\rho_s \sim \rho_e$ and therefore $\varepsilon \lesssim 1$. It can also be violated when the body is porous (ρ_s is small) or a dusty gas ($\rho_e \sim \rho_s$).

When $\varepsilon \lesssim 1$ and $Q = 0$, we shall write system (1.4) in the form

$$\frac{dx^*}{d\tau_2} = -x^* \exp\left(-\frac{1}{y^*}\right) + l(x_0 - x^*) \tag{2.2}$$

$$\frac{dy^*}{d\tau_2} = x^* \exp\left(-\frac{1}{y^*}\right) + m(y_0 - y^*)$$

$$x^* = \frac{\varepsilon R q_2 C_0 C_w}{E c_p}, \quad y^* = \frac{RT_w}{E_2}, \quad x_0 = \frac{\varepsilon R q_2 C_0}{E_2 c_p}, \quad y_0 = \frac{R}{E_2}\left(T_{sn} + \frac{q_1}{c_s}\right),$$

$$\tau_2 = \frac{k_2 t}{2}, \quad l = \frac{2\,\mathrm{Sc}\, f_w^2 \beta_x}{k_2}, \quad m = \varepsilon L_1 l$$

We know /5/ that the qualitative properties of the solutions of system (2.2) depend essentially on the magnitude of the parameter $L_2 = l/m$ characterizing the ratio of the rates of mass and heat transfer. When $L_2 < 1$ (a homogeneous reactor), the oscillatory instability of the stationary solutions and selfexcited oscillations are possible, while when $L_2 \geqslant 1$ (a reactor with fluidized catalyst bed) the non-stationary phenomena indicated are not possible, and a stable aperiodic stationary mode (or two such modes) will always exist.

The direct influence of the Lewis number L on the stability of combustion is opposite, in the present problem, to that experienced in the case of homogenous combustion /1-3, 14/. If the thermokinetic oscillations resulting from the thermodiffusive instability of the flame are possible in a homogeneous system only when $L < 1$ /1/, then selfexcited oscillations in the problem in question dealing with intense gasification may appear only when $L_1 = L\pi_c > 1$, since the necessary condition of their existence $L_2 < 1$ has the form $L_1 > 1/\varepsilon$ or $\varepsilon > \varepsilon_* = 1/L_1$ where $\varepsilon \lesssim 1$. This can obviously be explained by the fact that in this case the material controlling the rate of gas-phase reaction near the body is not brought in by diffusion, which causes the removal of material from the combustion zone into the boundary layer, but by injection resulting from the gasification process.

Since for many gases $L_1 \sim 1$ and $\varepsilon < 1$, it follows that in the majority of cases of interest from the practical point of view $L_2 = 1/(\varepsilon L_1) > 1$, and system (2.2), analogous to the model of a reactor with a fluidized catalyst layer /5/, will always have a stable, a periodic stationary solution.

Formally, when $L_1 > 1$ and ε increases (for example when the gas pressure or porosity of the body increases, or when the material disperses), ε can pass through its critical value ε_*, which leads to the appearance in system (2.2) of characteristic features of the model of a homogeneous continuous-action reactor /5/, and hence to the possibility of oscillatory instability and selfexcited oscillations. However, for the majority of real reactive media, this passage is impossible since the value of $\pi_\rho^* = (1 - \varepsilon_*)/\varepsilon_*$ corresponding to the value of ε_* is often practically impossible to attain (e.g. for petrol vapours $L_1 \approx 4$ and $\pi_\rho \approx 3$). It is, however, interesting that the condition $L_2 < 1$, which is practically impossible to satisfy when $\lambda_s \neq 0$, becomes, when the model of a thermally insulated wall ($\lambda_s = 0$) is used, an easily satisfied condition $L_1 > 1$ since here we have $\varepsilon = 1$ and $L = 1/L_1$.

Thus in the system in question without radiant heat exchange, under real conditions $(0 < \varepsilon \ll 1)$, we find that in case of the usual moderate gas pressures the mechanism of Frank-Kamenetskii /12/ excitation of thermokinetic oscillations characteristic for the homogeneous and some of the heterogeneous systems /1-4, 10, 11, 14/ does not function. It is suppressed by the simultaneous stabilizing influence of intense injection and conjugated heat exchange.

When $L_2 = 1$ $(\varepsilon L_1 = 1)$, system (1.4) will be analogous to the system describing a reactor with a temperature regulator /5/

$$\frac{dC_w}{d\tau_3} = 1 - C_w - W(C_w, \Theta_N) \tag{2.3}$$

$$\frac{d\Theta_N}{d\tau_3} = \Theta_{0N} - \Theta_N + W(C_w, \Theta_N) - U(\Theta_N)$$

$$W = \frac{1}{a}\bar{C}_1 C_w \exp\left(-\frac{EL_1}{\bar{C}_2\Theta_N}\right), \quad U = \frac{Q}{a\bar{C}_2}\left[\left(\frac{\bar{C}_2}{L_1}\right)^4 \Theta_N^4 - 1\right]$$

$$\Theta_N = L_1\Theta_{1w}/\bar{C}_2, \quad \Theta_{0N} = L_1\Theta_0/\bar{C}_2, \quad \tau_3 = a\tau_1$$

The necessary condition for the existence of selfexcited oscillations and oscillatory instability /2, 5/ for the system (2.3) takes the form $U'\partial W/\partial C_w > 1$. From this it follows that when there is no radiant heat exchange $(U = 0)$ or when there is radiation from the gas but not from the wall $(U = \text{const})$ when $U' = 0$, the oscillatory instability and selfexcited oscillations with mild excitation are impossible in system (1.4) when $L_2 = 1$. However, when the radiant flux from the surface is taken into account, $U' > 0$ and the necessary condition stated above becomes impossible to satisfy. Taking into account the radiation from the surface increases the domain of instability. The condition becomes possible when $L_2 = 1$, while when $Q = 0$, instability is possible only when $L_2 < 1$.

Applying the Routh-Hurwitz conditions /5/ to system (1.4) we can obtain the necessary condition of instability of the stationary modes

$$L_2 < L_2^* \equiv a\,(1 + 4Q\Theta_{1w}^{\circ 3}/(L_1 a)) \tag{2.4}$$

where Θ_{1w}° is the stationary value of Θ_{1w}. When condition (2.4) holds, selfexcited oscillations and oscillatory instability in the system become possible /2, 5/. The form of the expression for L_2^* in (2.4) confirms the conclusion made above for $L_2 = 1$ which also agrees with the results obtained in /11/, stating that the presence of radiation from the surface increases the domain of selfoscillations. Indeed, in the presence of radiation $(Q \neq 0)$ the necessary condition of instability (2.4) is less strict than in its absence $(Q = 0)$ when $L_2^* = a$. Thus the radiation represents a factor which destabilizes the behaviour of the system in question.

The results obtained here should be taken into account in calculating the combustion processes of strongly gasifying condensed materials, and in interpreting the results of the corresponding experiments. Using the dynamic systems described above and the methods of the analytic theory of differential equations /5, 13/, we can easily obtain the critical conditions of ignition and extinction, and the condition for selfexcited oscillations to occour /2-5, 10-12/.

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ASYMPTOTIC ANALYSIS OF THREE-DIMENSIONAL DYNAMIC EQUATIONS FOR THIN TWO-LAYER ELASTIC PLATES*

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In accordance with the method described in /1-3/, a derivation of two-dimensional equations of motion is given for a thin two-layer (non-symmetric) elastic plate. The mean values of the bending stiffness, the density, and Poisson's ratio are found, and the position of the middle plane is determined. In the coordinate system attached to this plane, the system of equations is separated into quasistatic equations for the longitudinal motion and a dynamic equation (of the ordinary kind) for the transverse component of the displacement. Unlike /1-3/, only one characteristic dimension in the longitudinal direction is introduced, which turns out to be sufficient and simplifies the analysis. Formulae of the complete field of stresses are provided. Stresses, which are of secondary importance for homogeneous plates, may be essential when the strength of the joint of the layers is considered.

1. Formulation of the problem. We shall consider a two-layer occupying a domain that is bounded or unbounded (in one or both directions). We denote by $h_i, \rho_i, E_i,$ and ν_i the thickness, the density of the material and the elastic characteristics of the upper layer $(i = 1)$ and lower layer $(i = 2)$. We choose an orthogonal system of coordinates as shown in the figure. The xy-plane is parallel to the plane of the plate and the values $z = z_0,\ z_1,\ z_2$ determine the plane of complete contact of the layers and the face planes of the plate. On these boundaries we impose the following conditions:

$$\begin{gathered}\tau_{\alpha z}^{(i)} = \tau_{\alpha *}^{(i)}(\xi, \eta, \tau), \quad z = z_i \quad (\alpha = \xi, \eta, \zeta) \\ \tau_{\alpha z}^{(1)} = \tau_{\alpha z}^{(2)}, \quad \mathbf{V}^{(1)} = \mathbf{V}^{(2)}, \quad z = z_0\end{gathered} \tag{1.1}$$

where $\tau_{\alpha\beta}$ and $\mathbf{V} = (v_\xi, v_\eta, v_\zeta)$ are the dimensionless components of the stress tensor and the displacement vector, and $\tau_{\alpha *}^{(i)}$ are given fairly smooth functions of the longitudinal coordinates and time τ. We use different normalization of the functions and different scale extension for different directions:

$$\begin{gathered}\sigma_{\alpha\beta} = E_i \tau_{\alpha\beta}, \quad \mathbf{u} = h\mathbf{V}, \quad 2h = h_1 + h_2 \\ (x, y) = l(\xi, \eta),\ z = h\zeta,\ t = t_0\tau,\ \varepsilon = h/l\end{gathered}$$

Here l is the least characteristic linear dimension of the pattern of deformation in the longitudinal direction, and t_0 is the characteristic time defined as follows:

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